

Yu. N. Alekseev and I. D. Zheltukhin

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 5, pp. 115-118, 1968

ABSTRACT: The pressures registered by a Pitot tube in uniform flows of aqueous polyacrylamide solutions have been measured. The tube readings are found to depend on the dimensions of the tube tip, the flow velocity, and the concentration and temperature of the solution.

In [1-3] it was noted that in the pipe flow of aqueous solutions of certain high-molecular compounds the velocity profile is measured with an error. This is indicated by the difference in the values of the flow rate determined by direct measurement and calculated from the measured velocity profile. Apparently, a similar effect has been observed in connection with velocity measurements in a submerged turbulent jet[4]; the jet momentum, calculated from the measured velocity profile for a polymer solution, proved to be 10-30% less than the value in the initial section of the jet.

The study of this effect is of considerable importance, since, according to the data of [1], it is observed only in solutions of those polymers which when added to a liquid reduce the turbulent drag.

A number of investigators [1-3, 5] associate the velocity measuring error in flows of the boundary layer type (such as pipe flow, jet flow, etc.) with a displacement of the mean velocity of the flow impinging on the Pitot tube and the appearance in polymer solutions of normal stresses produced by this shift.

Below, we present certain experimental results which indicate that even in the absence of such a shift, Pitot-tube velocity measurements in aqueous solutions of high-molecular compounds involve an error. The value of the ratio $k = H/H_0$, where H is the velocity head measured with the Pitot tube and H_0 the true velocity head, depends on the dimensions of the tip of the tube, the value of H_0 , the concentration of the solution C , and the temperature t .

The measurements were made in the rectangular working section of a hydrodynamic tube filled with test liquid (water or an aqueous polymer solution). The dimensions of the working section were 80 x 125 mm, its length about 500 mm. The capacity of the tube was about 100 l.

Velocity measurements with water flowing through the working section of the tube showed that in the flow core the nonuniformity of the velocity field did not exceed 1%. This made it possible to measure the velocity simultaneously with five Pitot tubes of different size arranged in a row at 15-mm intervals over the width of the working section.

The velocities were measured with ordinary flattened-tip tubes.

We have tabulated the data on the rectangular cross sections of these five tubes: height h and width b of tip, height h_0 and width b_0 of hole.

The values of the velocity head H were measured with a multi-tube manometer, connected with the Pitot tubes, as the difference between the pressure registered by each of the tubes and the static pressure, sampled through a 0.4-mm tap in the bottom wall of the tube.

By means of preliminary experiments conducted to measure the flow rate by means of Venturi tubes at contractions of from 1:2 to 1:4 it was established that in the investigated range of concentrations

N ₀	h , mm	h_0 , mm	b , mm	b_0 , mm
1	0.32	0.60	2.53	2.83
2	0.54	0.82	2.51	2.75
3	0.91	1.50	4.14	5.36
4	2.32	3.10	4.63	6.02
5	0.17	0.61	2.57	2.91

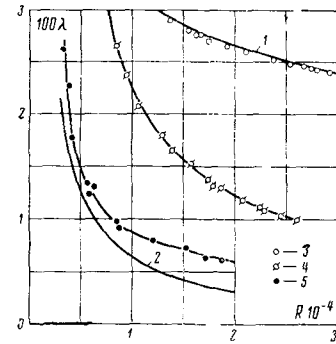


Fig. 1

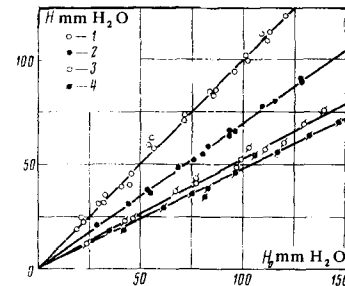


Fig. 2

the pressure drop at the flowmeter is a function only of the rate of flow of liquid through it and does not depend on the concentration of the solution. Hence, the true values of the velocity head H_0 in the working section of the tube were determined for the solutions from the dependence of H_0 on the pressure drop at a contraction of 1:3.4 established for water. The range of variation of H_0 was from zero to 150 mm H₂O, which corresponded to a variation of the velocity in the working section from 0 to 1.7 m/sec.

The measurements were made at two values of the flow temperature: 19 and 38° C. The deviation of the temperature from the indicated values under the experimental conditions did not exceed 2°.

As the polymer we used domestic polyacrylamide with a weighted-mean molecular weight of about $5 \cdot 10^6$. We tested aqueous solutions of five different polymer concentrations (by weight): $0.85 \cdot 10^{-4}$, $1.55 \cdot 10^{-4}$, $2.10 \cdot 10^{-4}$, $2.75 \cdot 10^{-4}$, and $3.40 \cdot 10^{-4}$. The corresponding values of the relative viscosity (viscosity of the solution referred to the viscosity of water at the same temperature) were 1.22, 1.42, 1.60, 1.83, and 2.10.

Under conditions of steady laminar flow aqueous solutions of polyacrylamide behave like Newtonian liquids: their viscosity does not depend on shear rate.

In turbulent motion the behavior of the polyacrylamide solutions becomes anomalous; namely, under similar conditions their turbulent drag is less than for water.

In Fig. 1 the resistance coefficient λ is plotted as a function of the Reynolds number R on the basis of measurements on water and aqueous solutions of polyacrylamide flowing in a circular tube 7.2 mm in diameter. Curves 1 and 2 correspond to the relations

$$100\lambda = 31.64/R^{0.25}, \quad 100\lambda = 6400/R.$$

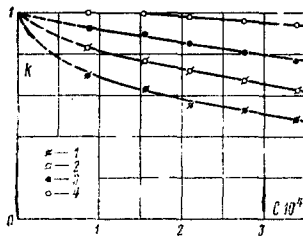


Fig. 3

Points 3, 4, and 5 correspond to concentrations $C = 0, 1.03 \cdot 10^{-4}$, and $4.5 \cdot 10^{-4}$. Hence it follows that adding polymer to the water leads to a reduction in the drag.

In Fig. 2 we have plotted the velocity head H measured with a Pitot tube as a function of the velocity head, using the data of measurements made with tube 1 in polyacrylamide solutions of varying concentration. Points 1, 2, 3, and 4 correspond to concentrations $C = 0.85 \cdot 10^{-4}, 2.75 \cdot 10^{-4}$, and $3.4 \cdot 10^{-4}$.

From these data it follows that if the dimensions of the tip of the tube are fixed, the registered pressure is the smaller, the higher the polymer concentration. In the velocity range investigated the relation $H(H_0)$, obtained for tubes of different size in solutions of varying concentration, is a linear one, which makes it possible to characterize the error in measuring velocity with Pitot tubes by means of the parameter $k = H/H_0$. In Fig. 3 we have plotted the parameter k as a function of the concentration of the solution at fixed temperature; curves 1-4 were constructed via the measurements made with tubes 1-4, respectively (for tip dimensions see table). As follows from Fig. 3, the pressure registered by the tube is the smaller, the higher the concentration of polymer in the solution and the smaller the dimensions of the tube tip. The data in Fig. 4 represent the results of measurements with tubes of like tip dimension but different hole dimension (tube 1 corresponds to curve 1 and tube 5 to curve 2). From these data it follows that the pressure registered by a Pitot tube in the solution is determined not by the dimensions of the hole, but by the dimensions and shape of the tip. In Fig. 5 the parameter k is shown as a function of the concentration for a solution with temperature $t = 19^\circ \text{C}$ (curve 1) and a solution with $t = 38^\circ \text{C}$, on the basis of measurements made with tube 1. Hence it follows that as the temperature increases the error in determining the velocity head decreases.

As a result of our investigation it has been established that using Pitot tubes to measure velocity leads to values that are too low as compared with the true values. Hence, before making measurements in a flow with a nonuniform velocity distribution (for example, in a pipe or a boundary layer), it is first necessary to calibrate the Pitot tube employed in a uniform flow.

The reason for the deviation of the total head measured by the tube from the true value is not clear. It is probable that the deviation is caused by the appearance of additional viscoelastic stresses that develop in the liquid near the tip of the tube when the flow is sharply decelerated. This is confirmed by the following qualitative considerations.

Taking into account the effect of viscous stresses on the Pitot tube readings mentioned in [6], we will examine the equations of liquid motion in terms of the stress components along the x -axis, which coincides with the critical streamline:

$$\rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \frac{\partial \sigma}{\partial x} \quad (1)$$

Here, ρ is the density of the liquid, u is the velocity component along the x -axis, p is the static pressure, and σ are the stresses due, in this

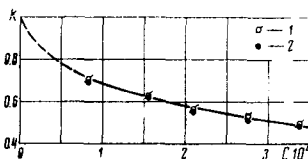


Fig. 4

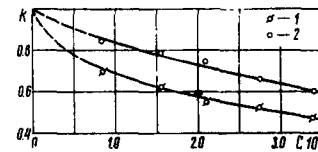


Fig. 5

case, to both the viscosity and the elasticity of the liquid. Integrating both sides of Eq. (1) from $x = 0$ to $x = \delta$ (δ is the thickness of the boundary layer near the stagnation point), we obtain

$$p_0 = (p_* + \frac{1}{2}\rho U_*^2) - (\sigma_* - \sigma_0) \quad (2)$$

Here, p_0 is the pressure registered by the tube; σ_0 are the additional stresses at the stagnation point at $x = 0$. An asterisk denotes values at $x = \delta$. At small shear rates the additional stresses due to the effect of the elastic properties of the liquid are proportional [7] to their square or, in our case, to the quantity $(\partial u / \partial x)_*^2$.

Neglecting the contribution of the viscous stresses and recalling that $\partial u / \partial x = 0$ at $x = 0$, we obtain

$$\sigma_* - \sigma_0 \approx \sigma_* \sim \left(\frac{\partial u}{\partial x}\right)_*^2 \quad (3)$$

where \sim is the sign of proportionality.

The value of $(\partial u / \partial x)_*^2$ can be determined [6] from the theory of potential flow over the tip of a Pitot tube and in order of magnitude

$$\left(\frac{\partial u}{\partial x}\right)_*^2 \sim \frac{U_0^2}{l^2} \quad (4)$$

Here, l is the characteristic dimension of the tube tip. Using (3) and (4), for the stress σ_* we have

$$\sigma_* = \beta H_0 \quad (5)$$

Here, H_0 is the velocity head, and β a parameter depending on the elastic characteristics of the liquid and the flow conditions near the tip of the tube. Subtracting the static pressure from both sides of Eq. (2) and using (5), we find that the relation between the previously introduced parameter k and β is given by

$$k = H / H_0 = 1 - \beta \quad (6)$$

From (4) and (5) it follows that at low velocities, when condition (3) is satisfied, the quantity k does not depend on velocity and the smaller the characteristic dimension l of the Pitot tube, the smaller k , which is confirmed by the experiments described.

REFERENCES

1. G. Astarita and L. Nicodemo, "Velocity distributions and normal stresses in viscoelastic turbulent pipe flow," *A.I.Ch.E. Journal*, vol. 12, no. 3, pp. 478-484, 1966.
2. K. A. Smith, E. W. Merrill, H. S. Mickley, and P. S. Virk, "Anomalous pitot tube and hot-film measurements in dilute polymer solutions," *Chem. Engn. Sci.*, vol. 22, no. 4, pp. 619-626, 1967.
3. A. B. Metzner and G. Astarita, "External flows of viscoelastic materials: fluid property restrictions on the use of velocity-sensitive probes," *A.I.Ch.E. Journal*, vol. 13, no. 3, May, pp. 550-555, 1967.
4. D. N. Jackley, "Drag-reducing fluids in a free turbulent jet," *International Shipbuilding Progress*, vol. 14, no. 152, 1967.
5. J. G. Savins, "A pitot tube method for measuring the first normal stress difference and its influence on laminar velocity profile determinations," *A.I.Ch.E. J.*, vol. 11, no. 4, pp. 673-677, 1965.
6. P. L. Chambre and S. A. Schaaf, "Pressure measurement," collection: *Physical Measurements in Gasdynamics and in Combustion* [in Russian], IL, Moscow, 1957.
7. R. Roscoe, "On the rheology of a suspension of viscoelastic spheres in a viscoliquid," *J. Fluid Mech.*, vol. 28, pp. 2, 273-293, 1967.

18 March 1968

Leningrad